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Some Effects of Mechanical Stress on the Breakdown
Voltage of p-n Junctions*

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Various factors determining stress induced changes in the breakdown voltage of Ge and Si p-n junctions are discussed. A model is developed which accounts for the multivalley band structure of semiconductors such as Ge and Si. Analytic expressions are developed for the change in breakdown voltage, $\Delta V/V_B$, as a function of a general stress. For Si, a linear decrease in $\Delta V/V_B$ with increasing stress is predicted. The proportionality factor is of the same order of magnitude as the band gap dependence on hydrostatic pressure. For Ge, an initial increase in $\Delta V/V_B$ followed by a decrease at high stress levels is indicated. The model is shown to be consistent with reported experimental data.

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I. INTRODUCTION

Several investigations have shown that mechanical stress can induce reversible changes in the electrical characteristics of p-n junctions.¹⁻⁵ In these investigations, large anisotropic stresses have been introduced into the junctions of diodes and transistors resulting in large changes in saturation currents of diodes and in current-gain for transistors. These changes have been attributed to stress induced changes in the energy band structure of the material and in particular to changes in the band gap.⁶ Recent experimental investigations have shown that the reverse breakdown voltage in Si and Ge diodes is also stress sensitive.^{7,8} The breakdown voltage in Si is found to be linearly related to uniaxial compression stress, while in Ge, the breakdown voltage is found to be a more complex function of the stress. The stress coefficient in Si is approximately equal to the coefficient found for band gap lowering under hydrostatic pressure. In Ge, the breakdown voltage is found to increase and then to decrease as stress increases.

The purpose here is to discuss the effects of multiple energy bands on the stress dependence of breakdown and to evaluate the order of magnitude of these stress induced changes in the breakdown voltage of Ge and Si diodes. A theoretical discussion is given which is based on Shockley's "Simple Model for Secondary Ionization"⁹ and the stress induced changes in the energy band structure of semiconductors.

II. THEORY

A. Simple Ionization Model

The ionization model to be discussed here for the unstressed junction was introduced by Shockley.⁹ The model is empirical in that it involves four-parameters, three of which are adjustable. It includes neither the energy band structure for energies greater than 1 ev from the band edges nor the effective masses of hot carriers. However, the model is found to be in reasonable agreement with experiment.

In the unstressed crystal, the four parameters of the model are as follows:

E_R = Energy of the "Raman" vibration mode.

L_R = Mean-free-path between scattering by "Raman" modes.

E_i = Threshold energy measured from the band edge above which a carrier may produce a hole-electron pair.

L_i = Mean-free-path between ionizations for a carrier with energy greater than E_i .

Using these parameters, an electron with energy greater than E_i generates on the average L_i/L_R phonons per ionization. In the low field limit, which has been shown to be valid for Ge and Si, the average number of scattering events, C , per ionization is

$$C = \frac{L_i}{L_R} \exp(E_i/qL_R F) , \quad (1)$$

where F is the electric field. The average energy, E_i^* , gained from the field by the carrier per ionization is then

$$E_i^* = \frac{L_i E_R}{L_R} \exp(E_i/qL_R F), \quad (2)$$

or

$$E_i^* = a \exp(b/F), \quad (3)$$

where a and b are constants of the material.

The secondary multiplication coefficient, $\alpha(F)$, is

$$\alpha(F) = \frac{qF}{E_i^*} = \frac{qFL_R}{L_i E_R} \exp(-E_i/qL_R F). \quad (4)$$

To extend the model to a stressed semiconductor, the effects of multiple conduction and valence levels on the ionization process must be considered.

B. Strain Dependent Ionization Theory

Mechanical strain has the effect of altering the energy band structure of semiconductors. The effects of strain on the conduction and valence band edge points for Ge and Si are reviewed in Appendix A. Under a general strain, the valence band edge not only shifts in energy, but also splits into two levels. The conduction band edge points also shift in energy both relative to the valence levels and to each other. The net results are different energy gaps depending on which valence and conduction band edge points are used to calculate the gap.

In the unstressed crystal, it is not necessary to know the exact dependance of the ionization energy on the energy band structure since this can be determined experimentally. Shockley assumes that for Si, E_i is equal to the band gap.⁹ Most investigators,^{10,11} however, find a better fit of theory to experimental data using an E_i slightly

larger than the band gap. This is particularly true in Ge. To obtain the numerical results for the effect of stress on breakdown voltage, a model which relates the ionization energy to the energies of the various bands is required. Such a model is discussed in a later section. For the present general development more general assumptions about the ionization process are sufficient.

Consider electron ionization first. In this ionization process, a hot electron in one of the conduction minima ($\langle 111 \rangle$ direction minima in Ge and $\langle 100 \rangle$ direction minima in Si) excites an electron from one of the valence bands to one of the conduction levels. The result being the creation of an electron-hole pair. The ionization threshold energy is the minimum energy at which the above process can occur. This energy is expected to depend upon the shape of the energy bands in which the ionizing electron and the created pair are located as well as the band edge point energies of the various bands. In the unstressed crystal, all the conduction minima are located at the same energy so there can be only two different threshold energies--one associated with the heavy hole band and one associated with the light hole band. With a stress condition, the conduction levels are split in energy and consequently the threshold energies for the various nondegenerate conduction levels should be different.

Here it is assumed that the ionization threshold energy depends upon the conduction level in which the ionizing hot electron is located and the valence level in which the created hole is located. This assumes that the created electron is located either in the same

energy minimum as the hot electron or is located in the equivalent energy minimum in the opposite direction in k-space. This is a reasonable assumption, since crystal momentum can be more easily conserved in this process.¹²

An ionization threshold energy E_{nm} can then be associated with each of the combinations of valence and conduction levels, where n denotes the conduction level and m denotes the valence level. There are six such combinations for Si and eight for Ge. Each of the ionization processes may also have a different mean free path denoted by L_{nm} .

It will be assumed here that L_R and E_R are not functions of stress, and that L_R is the same for all the conduction band minima.

Using the above definitions, the number of ionizations which leaves a hole in the valence level m and an electron in conduction level n per scattering event, is denoted by $1/C_{nm}$. From Eq. (1) this is

$$\frac{1}{C_{nm}} = \frac{L_R}{L_{nm}} \exp(-E_{nm}/qL_R F) . \quad (5)$$

The total number of ionizations per scattering event is obtained by summing over the ionization processes for a hot electron in a given conduction minimum and then averaging over the conduction levels. In averaging over the conduction levels, $1/C_{nm}$ must be multiplied by the fraction of hot electrons $n_{hn}/\sum_n n_{hn}$, in each conduction minimum. The average number of ionizations per scattering event is then

$$\frac{1}{C} = \frac{\sum_{n=1}^{\beta} n_{hn} \sum_{m=1}^{\gamma} \frac{L_R}{L_{nm}} \exp(-E_{nm}/qL_R F)}{\sum_{n=1}^{\beta} n_{hn}}, \quad (6)$$

where γ is the number of valence levels and β is the number of conduction levels.

The evaluation of the above formal expression requires an assumption on the distribution of hot electrons among the conduction levels. This distribution of hot electrons depends upon the importance of intervalley and intravalley scattering. If intervalley scattering is negligible, the conduction minima can be considered as independent of each other and

$$n_{hn} \propto \exp(-E_{cn}/kT), \quad (7)$$

where E_{cn} is the band edge energy of the conduction level. This is just the equilibrium distribution of electrons among the valleys. On the other hand, if intervalley scattering is predominate, the electrons are more uniformly distributed among the levels because of the increased effective temperature of the hot electrons. On the basis of Shockley's model for ionization, the hot electrons in a given conduction minima are distributed in energy according to the relationship

$$n(E) \propto \exp(-E/qL_R F). \quad (8)$$

When intervalley scattering dominates, one would also expect this distribution to hold for electrons among the conduction levels and in this case

$$n_{hn} \propto \exp(-E_{cn}/qL_R F) . \quad (9)$$

To keep the assumption more general, the hot electrons are assumed to be distributed among the valleys according to the relationship

$$n_{hn} \propto \exp(-E_{cn}/E_1) , \quad (10)$$

where E_1 equals kT and $qL_R F$ for the cases considered above. As is subsequently shown, the low stress case and the very high stress case are independent of the choice of E_1 .

The total number of ionizations per scattering event then becomes

$$\frac{1}{C} = \frac{\sum_{n=1}^{\beta} \exp(-E_{cn}/E_1) \sum_{m=1}^{\gamma} \frac{L_R}{L_{nm}} \exp(-E_{nm}/qL_R F)}{\sum_{n=1}^{\beta} \exp(-E_{cn}/E_1)} . \quad (11)$$

The secondary multiplication coefficient for electrons is then given by

$$\alpha(F) = \frac{qFL_R}{E_R} \frac{\sum_n \exp(-E_{cn}/E_1) \sum_m \frac{1}{L_{nm}} \exp(-E_{nm}/qL_R F)}{\sum_n \exp(-E_{cn}/E_1)} . \quad (12)$$

The value of L_{nm} in Eq. (12) may be different for holes created in the lower valence level as compared to holes created in the upper valence level. One would also expect L_{nm} to be slightly dependent upon the direction of the applied field. For example, if the applied field is along a conduction valley direction, one expects a smaller value of L_{nm} for an electron created in the energy minimum along the field direction than for the other energy minimum. The difference between these values depends upon intervalley and intravalley scattering

of hot electrons. If there is sufficient scattering to produce essentially a random distribution of the hot electrons both in and among the conduction valleys, L_{nm} should be essentially the same for all valleys. Sufficient information is not available to permit a detailed evaluation of this parameter. It is also noted that L_{nm} is a linear factor influencing $\alpha(F)$, while the ionization energy E_{nm} is exponentially related to $\alpha(F)$. Thus small changes in E_{nm} tend to be more important in determining $\alpha(F)$ than small changes in L_{nm} . It will therefore be assumed that L_{nm} is the same for all ionization processes. This assumption makes the stress dependence of the secondary multiplication coefficient independent of the direction of the electric field.

For Ge with an electric field in the [100] direction, the component of electric field along each of the valley directions is the same, and the assumption of a constant L_{nm} for all the valleys should be valid. The assumption should be least valid for a field in the [111] direction. For Si, the assumption should be most accurate for a field in the [111] direction and least accurate for a field in the [100] direction.

Assuming that L_{nm} is constant, it can readily be seen from Eq. (12) by considering the unstressed case ($E_{nm} = E_i$) that

$$L_{nm} = \gamma L_i . \quad (13)$$

Equation (12) becomes

$$\alpha(F) = \frac{qFL_R}{E_R L_i \gamma} \frac{\sum_n \sum_m \exp(-E_{cn}/E_1 - E_{nm}/qL_R F)}{\sum_n \exp(-E_{cn}/E_1)} . \quad (14)$$

This is the basic equation used to describe stress dependent voltage breakdown, when electron ionization is the dominant process.

Although the concepts for hole ionization are similar to electron ionization, there are important differences in the details. If the difference in effective mass for the two hole bands is neglected and the ionization energy is assumed to depend only upon the band on which the hot hole exists and upon the conduction minimum to which the electron is excited, the evaluation of hole ionization is very similar to that for electron ionization. The difference is that the average number of ionizations per scattering event is obtained by summing over the conduction levels and averaging over the valence levels. This leads to the expression

$$\alpha(F) = \frac{qFL_R}{E_R L_i \beta} \frac{\sum_m \sum_n \exp(E_{vm}/E_1 - E_{nm}/qL_R F)}{\sum_m \exp(E_{vm}/E_1)}, \quad (15)$$

where E_{vm} is the energy of the valence levels.

The large difference in density of states or effective mass for the two valence bands, especially in Ge, makes the neglect of these differences in the above expression open to question. There are, however, factors which make the above expression a better approximation than it would at first appear. First, the average energy of the holes is on the order of E_R which is larger than the thermal energy and, for large energies, the density of states for the two bands become more equal. In Si the density of states are not too different for energies larger than about 0.02 ev.¹³ For Ge the energy is considerably larger. A second factor for unequal hole masses is that L_R is different for the

two bands. For a single spherical energy band, L_R has been shown to depend upon the mass as $m^{-2.14}$. This partially compensates for the difference in the density of states of the two bands. The compensating effects in the two bands can be thought of in the following manner. The holes in the "light" hole band are more rapidly accelerated than the "heavy" holes to energies sufficient to produce optical phonons or to produce ionizations. Thus, while there are fewer holes in the light hole band, they undergo scatterings and produce ionizations at a faster rate which tends to produce some compensation. Here the purpose is to discuss the major features of stress effects on breakdown so that the simple expression of Eq. (15) is sufficient.

C. Strain Dependent Breakdown

Even when the ionization coefficient is known, the calculation of the breakdown voltage of a p-n junction is difficult, especially when the ionization coefficients for holes and electrons are unequal.¹⁰ Including the effect of stress further complicates the calculation. A simplifying approximation is made to obtain a tractable model. Because of the exponential dependence of α upon $1/F$, the major contributions to α come from the depletion region near the maximum field point. Let α_m be the value of α at the maximum field point, and consider the effect of stress upon α_m . If the maximum junction field is unchanged when stress is applied (i.e. if the junction voltage is unchanged), the ionization coefficient changes and hence the current multiplication factor for the junction changes. On the other hand, if α_m is to remain

constant under stress, the maximum field must change. The approximation which is made here is that at a constant current multiplication factor for the junction, the maximum junction field is changed such that α_m remains constant. The following development is for electron ionization; the corresponding development for hole ionization is similar.

To obtain the change in maximum field required to keep α_m constant, let

$$E_{nm}(\sigma) = E_i + \Delta E_{nm}, \quad (16)$$

and

$$F = F_o + \Delta F, \quad (17)$$

where E_i and F_o are the unstressed values and $E_{nm}(\sigma)$ is the stress dependent ionization energy. Then for electron ionization neglecting second order effects,

$$\alpha_m(F_o + \Delta F) = \frac{qF_o}{E_R} \left(1 + \frac{\Delta F}{F_o}\right) \frac{L_R}{\gamma L_i} \exp[-E_i(1 - \Delta F/F_o)/qL_R F_o] \quad (18)$$

$$\times \frac{\sum_n \sum_m \exp(-\Delta E_{cn}/E_1 - \Delta E_{nm}/qL_R F_o)}{\sum_n \exp(-\Delta E_{cn}/E_1)}.$$

By equating this to the unstressed value of $\alpha_m(F_o)$, it is found that

$$1 = \left(1 + \frac{\Delta F}{F_o}\right) \frac{1}{\gamma} \exp(E_i \Delta F / qL_R F_o^2) \frac{\sum_n \sum_m \exp(-\Delta E_{cn}/E_1 - \Delta E_{nm}/qL_R F_o)}{\sum_n \exp(-\Delta E_{cn}/E_1)}. \quad (19)$$

This expression relates the change in the maximum junction electric field to the stress induced changes in the energy levels.

To obtain the change in junction voltage at a constant current multiplication factor, the change in maximum junction field must be

related to the junction voltage. The maximum field is proportional to $V^{1/2}$ for a step junction.¹⁵ Thus for small changes

$$\frac{\Delta V}{V_B} \approx 2 \frac{\Delta F}{F_o}, \quad (20)$$

where V_B is the unstressed breakdown voltage at a constant current level. For most p-n junctions, the step junction approximation is reasonably accurate at large reverse bias.

For small changes in the breakdown voltage ($\Delta V/V_B \ll 1$), Eq. (19) can be solved for the change in breakdown voltage to give

$$\frac{\Delta V}{V_B} = - \frac{2}{(1 + E_i/qL_R F_o)} \ln \left\{ \frac{1}{\gamma} \frac{\sum_n \sum_m \exp(-\Delta E_{cn}/E_1 - \Delta E_{nm}/qL_R F_o)}{\sum_n \exp(-\Delta E_{cn}/E_1)} \right\}. \quad (21)$$

For hole ionization a similar development leads to

$$\frac{\Delta V}{V_B} = - \frac{2}{(1 + E_i/qL_R F_o)} \ln \left\{ \frac{1}{\beta} \frac{\sum_n \sum_m \exp(\Delta E_{vm}/E_1 - \Delta E_{nm}/qL_R F_o)}{\sum_m \exp(\Delta E_{vm}/E_1)} \right\}. \quad (22)$$

In the low stress region ($\Delta E_{nm}/qL_R F_o \ll 1$ and $E_{cn}/E_1 \ll 1$), the equations for both electron and hole ionization simplify to

$$\frac{\Delta V}{V_B} = \frac{2}{qL_R F_o (1 + E_i/qL_R F_o)} \frac{1}{\gamma\beta} \sum_{n,m} \Delta E_{nm}. \quad (23)$$

In this limit, the factor $(1/\gamma\beta) \sum \Delta E_{nm}$ is the simple average of the changes in the ionization threshold energies.

To carry the development further and obtain numerical results requires a model for the changes in the ionization threshold energies with stress. One assumption is that the ionization threshold energy

is equal to the band gap. This implies that momentum is supplied by phonons. Neglecting phonon assisted processes the ionizing electron must have both enough energy to create the hole-electron pair (E_g) and also enough excess energy to conserve both momentum and energy between the single incident particle and the three final particles. For a direct band gap material the minimum energy is readily found to be $3E_g/2$. For an indirect band gap material such as Ge and Si, it has recently been shown¹² that a consideration of energy and crystal momentum conservation leads to an expression of the form

$$E_i \simeq k_1 + k_2 E_g, \quad (24)$$

where k_1 and k_2 are constants depending in rather complex ways on the effective masses of the three particles. The effective masses entering into k_1 and k_2 are not the band edge effective masses, but are the masses which give the best fit to the energy bands over the energy range from zero to the ionization energy. Based upon this model, the ionization threshold energy between a conduction level and a valence level is

$$E_{nm} = k_1 + k_2(E_{cn} - E_{vm}). \quad (25)$$

This is the model used in this work to relate the band structure to the ionization energy.

Stress can have two effects on E_{nm} as defined above through changes in the energy levels ($E_{cn} - E_{vm}$) and through changes in the shape (or effective mass) of the energy levels which results in changes in k_1 and k_2 . The effect of stress on the band edge energy levels is

known but the effect of stress on the effective mass of the bands especially at large energies has not been as thoroughly investigated. Some work has been done on the effective masses near the band edges, but the effective mass at large energies probably changes less than near the band edges, especially for the hole bands. Near the band edges there is no first order change in the effective mass with stress for electrons.^{16,17} Also under compression stress in the [100] and [111] directions there is no first order change in the effective mass of the lower valence level.¹⁸ Based upon the above facts, k_1 and k_2 have been taken as independent of stress for our first order model and the changes in ionization energy are taken to be

$$\Delta E_{nm} = k_2(\Delta E_{cn} - \Delta E_{vm}). \quad (26)$$

For equal hole and electron masses, k_2 is 0.59 for electron ionization in Ge and 0.63 for hole ionization in Ge and for both electron and hole ionization in Si.¹²

Using the above model for ionization, the change in breakdown voltage at low stress is given from Eq. 24 as

$$\frac{\Delta V}{V_B} = \frac{2k_2KP}{E_i} \left(1 + \frac{qL_R F_o}{E_i}\right)^{-1}, \quad (27)$$

where K is the coefficient relating the change in band gap to hydrostatic pressure and P is the hydrostatic component of stress.

Typical values of the parameters in Eq. (27) are listed in Table I. Using these values, Eq. (27) reduces to

TABLE I: Typical Values of K , F_o , L_R and E_i in Ge and Si

	$K(10^{-12} \text{ ev cm}^2/\text{dyne})$	$F_o(10^5 \text{ volts/cm})$	$L_R(\text{\AA}^\circ)$	$E_i(\text{ev})$
Si	-1.5^a	5^b	$50-70^d$	$1.3^{f,8}$
Ge	5.0^e	0.82^c	65	$1.0^f, 0.93^g$

a. W. Paul and G. L. Pearson, Phys. Rev., 98, 1755 (1955).

b. Ref. 9

c. R. Yee, J. Murphy, A. D. Kurtz, and H. Bernstein, J. Appl. Phys. 30, 596 (1959).

d. Ref. 10

e. P. W. Bridgman, Proc. Am. Acad. Arts Sci. 79, 129 (1951).

f. Values calculated for electron ionization, Ref. 12.

g. Values calculated for hole ionization, Ref. 12.

$$\text{Si: } \frac{\Delta V}{V_B} = -1.2 \times 10^{-12} P \text{ cm}^2/\text{dyne}, \quad (28)$$

$$\text{Ge: } \frac{\Delta V}{V_B} = 6.0 \times 10^{-12} P \text{ cm}^2/\text{dyne}. \quad (29)$$

As can be seen from Eq. (27), at low stress levels, $\Delta V/V_B$ is independent of stress orientation since only the hydrostatic component of stress remains in the expressions. It is significant that at low stress levels, $\Delta V/V_B$ has an opposite sign for Ge and Si.

Returning to Eq. (22), it is seen that in the high stress case, the minimum conduction level and the maximum valence level will predominate, i.e., the ΔE_{nm} with the largest negative value. The corresponding ΔE_{cn} is negative. In general for uniaxial compression stresses at least one of the ΔE_{nm} 's will be negative for both Ge and Si. Therefore, for high stresses, $\Delta V/V_B$ decreases for both materials, and is independent of the value of E_1 . This is true for both electron and hole ionization.

For a given value of the ratio $E_1/qL_R F_O$, the change in breakdown voltage under stress can be calculated from Eqs. (21) and (22) for electron and hole ionization respectively. To account for the increased effective temperature of the carriers in the high field, the value $E_1 = qL_R F_O$ has been used in most of the calculations. The following deformation potentials were used in evaluating the shifts in the energy levels (see Appendix A):

Ge	Si
$D_u = 3.15$	$D_u = 2.04$
$D'_u = 6.06$	$D'_u = 2.68$
$\Xi_u = 19.2$	$\Xi_u = 11$
$D_d - (\Xi_d + \Xi_u/3) = 4.82$	$D_d - (\Xi_d + \Xi_u/3) = -1.44$

The calculated changes in breakdown voltage in Ge with stress for electron and hole ionization are shown in Figs. 1 and 2 for uniaxial stresses in the [100], [111], and [110] directions. The curves are plotted in terms of a normalized stress and a normalized voltage change given by the expressions

$$\sigma' = (E_g / qL_R F_o) \sigma, \quad (30)$$

$$\frac{\Delta V'}{V_B} = \frac{1}{2} \left(1 + \frac{E_i}{qL_R F_o} \right) \frac{\Delta V}{V_B}.$$

The curves illustrate the increase in breakdown voltage in Ge at low stresses, independent of the stress orientation, and the decrease in breakdown voltage at large stress values. It is also noted that for electron ionization, the changes for the [100] direction require approximately an order of magnitude larger stress than the other two directions.

A comparison of the theoretical changes (for electron ionization) in breakdown voltage for two values of E_1 are shown in Fig. 3. The two values of E_1 ^{are} ~~and~~ kT and $qL_R F_o$. It is recalled that the value kT

the saturation current. The reason this is necessary is that the saturation current is also stress sensitive. The multiplication factor, M is related approximately to the applied voltage, V_A , by¹⁹

$$M = \frac{1}{1 - (V_A/V_B)^n}, \quad (31)$$

where n is a constant for the device. The diode current, I , is given by

$$I = I_s M, \quad (32)$$

where I_s is the reverse saturation current. Combining Eqs. (30) and (31) gives

$$V_A = V_B (1 - I_s/I)^{1/n}. \quad (33)$$

If $M \gg 1$,

$$V_A = V_B (1 - \frac{1}{n} \frac{I_s}{I}). \quad (34)$$

Neglecting second order terms, the relative change in the applied voltage at constant current is

$$\frac{\Delta V_A}{V_B} \approx \frac{\Delta V}{V_B} - \frac{\Delta I_s}{nI}. \quad (35)$$

The change in applied junction voltage is then less than the change in the breakdown voltage by the factor $V_B \Delta I_s / nI$. This is negligible only if nI is large compared with ΔI_s . Using Eq. (35), $\Delta V/V_B$ can be evaluated from an experimental plot of current versus voltage as a function of stress if ΔI_s is known as a function of stress. For a uniformly stressed junction in which the total junction area is stressed, $\Delta I_s/I_{s0}$ is given by⁶

$$\frac{\Delta I_s}{I_{so}} = \frac{1}{\gamma\beta} \sum \exp\left(\frac{\Delta E_{cn} - \Delta E_{vm}}{kT}\right) - 1 = (f(\sigma) - 1), \quad (36)$$

where I_{so} is the unstressed saturation current. This gives the following expression for $\Delta V_A/V_B$

$$\frac{\Delta V_A}{V_B} \approx \frac{\Delta V}{V_B} - \frac{(f(\sigma) - 1)}{nM_o}, \quad (37)$$

where M_o is the multiplication factor between the unstressed saturation current and the current at which V_A is measured.

In Si, it has been found experimentally that uniaxial compression stress causes a linear decrease in $\Delta V_A/V_B$ by the factor^{7,8}

$$\frac{\Delta V_A}{V_B} \approx (-1 \times 10^{-12} \text{ cm}^2/\text{dyne}) \sigma, \quad (38)$$

where σ is the magnitude of the uniaxial stress. The theoretical proportionality constant is $-0.4 \times 10^{-12} \text{ cm}^2/\text{dyne}$ from Eq. (28) where $\sigma = P/3$. The theoretical value was calculated using the value $-1.5 \times 10^{-12} \text{ ev cm}^2/\text{dyne}$ for the hydrostatic pressure dependance of the band gap. Although this value is normally used, there is some variation in it depending on the method of measurement and the stress level.²⁰ In fact, it has been shown that the coefficient is stress dependant and at high stress levels, it is approximately $-2.5 \times 10^{-12} \text{ ev cm}^2/\text{dyne}$.²¹ Using the latter value gives a proportionality constant in Eq. (28) of approximately $-0.67 \times 10^{-12} \text{ cm}^2/\text{dyne}$.

As predicted by the theory, very little if any variation in the phenomenon with crystal orientation is seen experimentally in Si.^{7,8}

Preliminary measurements on breakdown voltage changes with hydrostatic pressure have given results which were an order of magnitude smaller than those for uniaxial stress.⁸ The present theory predicts that the voltage changes induced by hydrostatic pressure should be three times those of uniaxial stresses. At the present time, there is no explanation for these results.

Rindner has made measurements of the breakdown voltage change as a function of uniaxial stress in Ge diodes.⁷ The experimental points in Fig. 4 are the results he obtained from a [110] and a [100] uniaxial stress. The solid curves in the figure were calculated using the theory for hole ionization. The hole ionization curves were used because the secondary multiplication factor for holes is larger in Ge. The normalizing factor $E_g/qL_R F_0$ was considered as an adjustable parameter in fitting the theory to experiment. The values needed to fit the experimental data are approximately 8.5 and 16 for the [110] and [100] directions respectively. These values are in good agreement with the value of 12.6 calculated from the data in Table I. Attempts to fit the theory to published experimental data taken for [111] oriented uniaxial stress⁷ have been less successful. It should be noted that the data can also be made to agree with the theory for electron ionization by a suitable choice of $L_R F_0$. For the [100] direction, however, an unreasonably small value of $L_R F_0$ is required to produce a good fit to the data. It is difficult to draw too many conclusions from these comparisons without knowing the repeatability of the experimental results and the conditions under which the data was obtained. As

shown in Fig. 4, the experimental results and theoretical curves deviate from each other for low stress levels. This could result from nonuniform breakdown in the junction due to microplasma and surface effects.

V. DISCUSSION

The theoretical model for stress induced changes in breakdown voltage is in generally good agreement with experiment. In view of the simple model used for breakdown and the assumptions necessary to arrive at an analytical expression for the effect, it is surprising that the theory agrees as well as it does. The fact that the theory predicts an orientation dependence in Ge but no such dependence in Si in agreement with the experimental observations, lends considerable support to the theory.

Care must be exercised when comparing the theory to experiment since most practical devices do not exhibit uniform breakdown. Also, it is difficult to determine what percentage of the multiplication factor results from hole ionization as compared to electron ionization. These factors are necessary to make accurate comparison between theory and experiment. The comparison for Ge made in the preceeding section was based on the assumption that the ionization energy is that of holes.

APPENDIX A

Both Ge and Si have multiple conduction minima in k-space. Ge has eight such minima which lie in the $\langle 111 \rangle$ direction and are located at the L_1 point in k-space while silicon has six conduction minima which occur in the $\langle 100 \rangle$ direction and are located approximately 85% of the distance from $k = (000)$ to the X_1 symmetry point. The maximum valence levels, Γ'_{25} , for both Ge and Si are located at $k = (000)$. The Γ'_{25} level is degenerate in energy. If the crystal is mechanically deformed, the crystal symmetry and the lattice spacings are altered and hence the energy bands change.

Herring and Vogt have considered the effect of mechanical strain on the conduction minima in both Ge and Si.²² The results of their work are summarized in Table II. The valley directions are identified by the subscripts on the conduction energy level changes ΔE_c 's. The E 's are deformation potential constants and the e 's are conventional strains. Table III lists some of the values of the deformation potential constants. The values appearing in brackets are theoretical values calculated by Kleinman, et al.²³⁻²⁵

The effects of mechanical strain on the valence levels of Ge and Si are much more complicated than the effects on the conduction levels. Kleiner and Roth have considered the effects of strain on the Hamiltonian of the valence band edge.²⁶ Diagonalizing their expression for the Hamiltonian gives the following expression for the change in energy of the valence level²⁷

$$\Delta E = D_d e \pm \left[\left(\frac{2}{3} D_u \right)^2 (e_1^2 + e_2^2 + e_3^2 - e_1 e_2 - e_1 e_3 - e_2 e_3) + \frac{1}{3} (D'_u)^2 (e_4^2 + e_5^2 + e_6^2) \right]^{1/2}. \quad (A1)$$

The D's appearing in the above expression are the valence band deformation potential constants. From Eq. (A1), it is seen that there is not only a shift of the level due to D_d , but also a splitting of the level due to D_u and D'_u , which removes the degeneracy. For convenience of notation, let the upper band, positive sign of Eq. (A1), be ΔE_{V1} and the lower band, negative sign, be ΔE_{V2} .

TABLE II. Equations describing the change in the band edge points in the conduction band of Ge and Si as a function of strain.

Valley Direction	Band Edge	Energy
Ge		
$[111], [\bar{1}\bar{1}\bar{1}]$	ΔE_{c1}	$(\Xi_d + \Xi_u/3)e + \Xi_u(e_4 + e_5 + e_6)/6$
$[1\bar{1}\bar{1}], [\bar{1}11]$	ΔE_{c2}	$(\Xi_d + \Xi_u/3)e + \Xi_u(e_4 - e_5 - e_6)/6$
$[1\bar{1}1], [\bar{1}1\bar{1}]$	ΔE_{c3}	$(\Xi_d + \Xi_u/3)e + \Xi_u(-e_4 + e_5 - e_6)/6$
$[11\bar{1}], [\bar{1}\bar{1}1]$	ΔE_{c4}	$(\Xi_d + \Xi_u/3)e + \Xi_u(-e_4 - e_5 + e_6)/6$
Si		
$[100], [\bar{1}00]$	ΔE_{c1}	$\Xi_d e + \Xi_u e_1$
$[010], [0\bar{1}0]$	ΔE_{c2}	$\Xi_d e + \Xi_u e_2$
$[001], [00\bar{1}]$	ΔE_{c3}	$\Xi_d e + \Xi_u e_3$

TABLE III. Deformation Potential Coefficients (ev/unit dilation) for Ge and Si. (Kleinman's theoretical values are shown in brackets.)

Coefficient	Si	Ge
D_d	[-2.09]	[-2.09]
D_u	2.04 ^a , [3.74]	3.15 ^b , [3.74]
D'_u	2.68 ^a , [4.23]	6.06 ^b , [3.6]
Ξ_d	[-4.99]	[-10.16]
Ξ_u	11 ^c , 8.3 ^d , [+9.6]	19.2 ^e , [11.4]
Ξ'_u	5.7 ^f	
$D_d - (\Xi_d + \frac{1}{3} \Xi_u)$	-1.44 ^g , [-0.30]	4.82 ^g , [4.27]

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APPENDIX B: Effect of Stress on V_o .

The applied voltage differs from the junction voltage in reversed biased junctions by the built-in potential, i.e.

$$V = V_A + V_o , \quad (B1)$$

where V is the junction voltage, V_A is the applied voltage, and V_o is the built-in voltage. Mechanical stress not only changes the junction voltage, but it also can change the built-in voltage. The relative change in the applied voltage at breakdown is

$$\frac{\Delta V_A}{V_B} = \frac{\Delta V}{V_B} - \frac{\Delta V_o}{V_B} . \quad (B2)$$

The built-in voltage for a step junction is

$$V_o = \frac{kT}{q} \ln \left(\frac{N_A N_D}{n_i^2} \right) , \quad (B3)$$

when N_A and N_D are the acceptor and donar concentrations on the p and n side of the junction respectively and n_i is the intrinsic carrier concentration ($n_i^2 = pn$).

Wortman, Hauser, and Burger have treated the effect of stress on p and n.⁶ Using their results and forming the pn product gives

$$n_i^2 = pn = p_o n_o f(\sigma) , \quad (B4)$$

when $p_o n_o$ is the unstressed intrinsic carrier concentration. The factor $f(\sigma)$ is given by

$$f(\sigma) = \frac{1}{\gamma\beta} \sum \exp\left(\frac{\Delta E_{cn} - \Delta E_{vm}}{kT}\right) . \quad (B5)$$

The change in V_o divided by V_B becomes

$$\frac{\Delta V_o}{V_B} = - \frac{kT}{qV_B} \ln f(\sigma) . \quad (B6)$$

At room temperature $qL_R F_o / E_g$ is approximately equal to kT for Ge so that

$$\frac{\Delta V_o}{V_B} \approx 4 \frac{E_i}{qV_B} \frac{\Delta V}{V_B} . \quad (B7)$$

Therefore, for Ge, if V_B is larger than several volts, $\Delta V_o / V_B$ can be neglected compared to $\Delta V / V_B$.

In Si, $qL_R F_o$ is approximately 10 kT so that, except for hydrostatic stresses, the exponential terms will contribute to the built-in potential at a lower stress level than they do for the junction voltage. Therefore, if V_B is not large, the change in the built-in potential can reduce the change in the applied voltage. In fact, if V_B is on the order of one volt, the built-in potential change can dominate the applied voltage change.

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* This work was supported in part by the National Aeronautics and Space Administration under Contract No. NASr-222.

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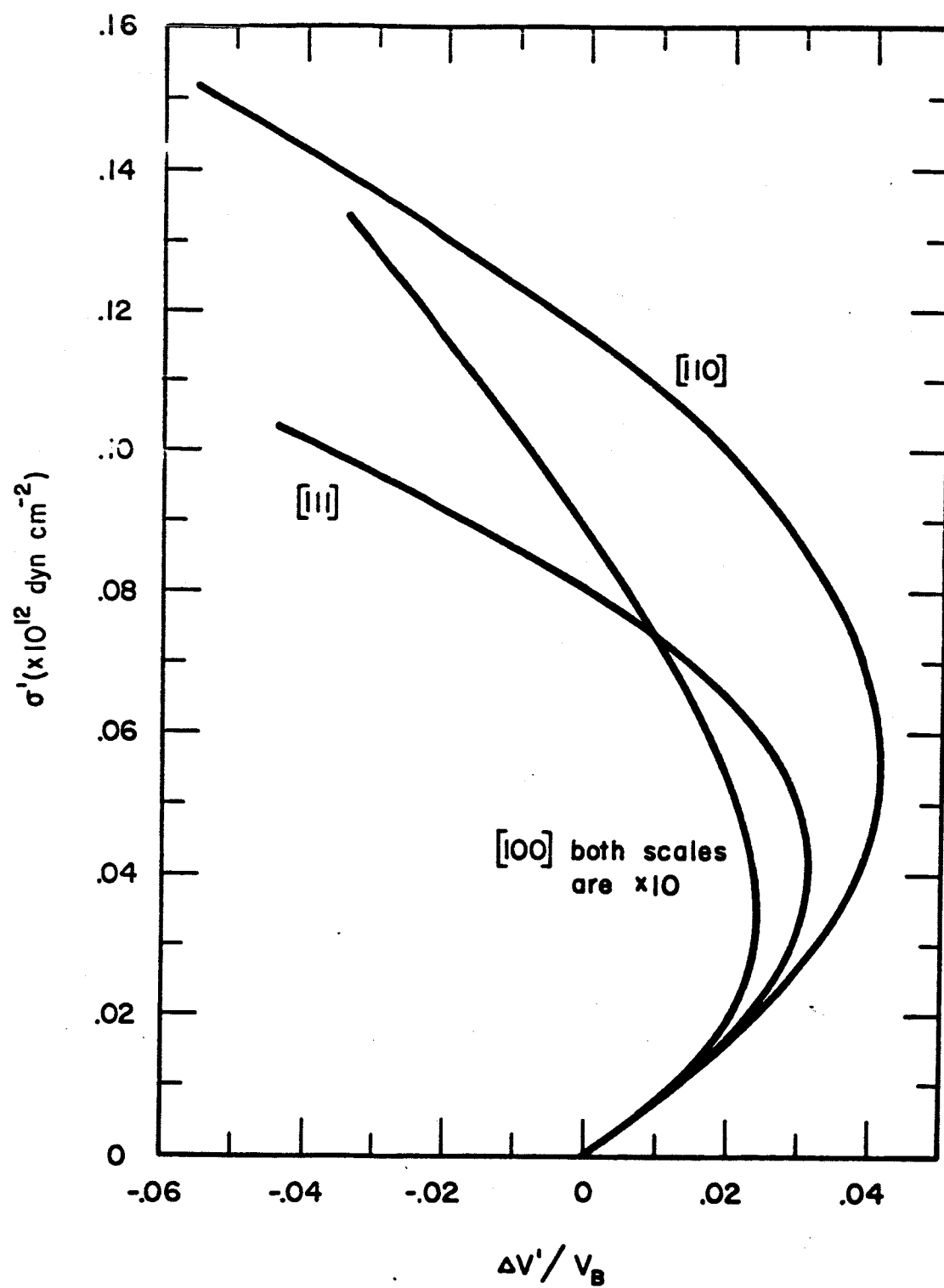


Figure 1. Theoretical values of $\Delta V'/V_B$ as a function of σ' for electrons with stress in the [100], [011], [111] directions in germanium.

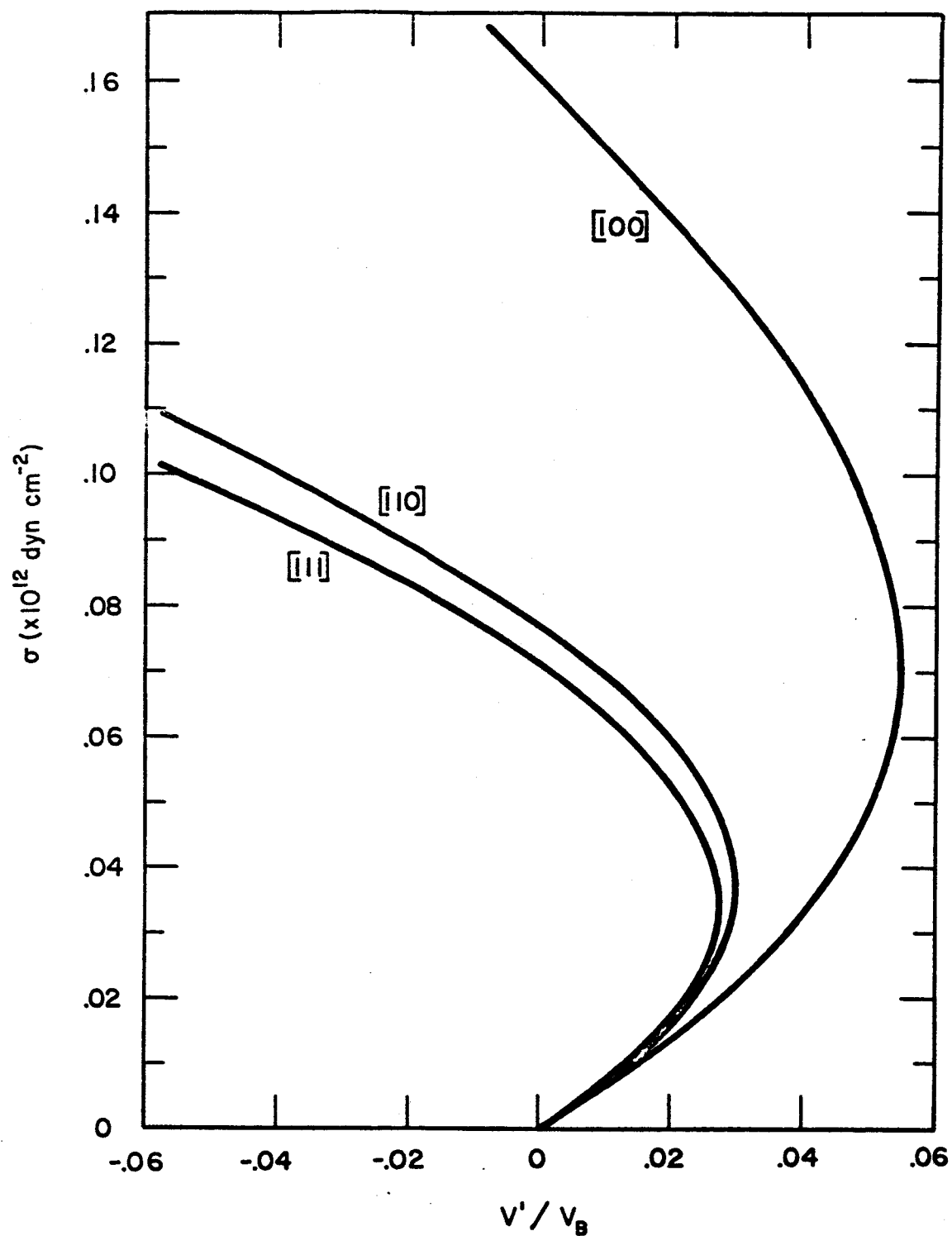


Figure 2. Theoretical values of $\Delta V'/V_B$ as a function of σ' for holes with stress in the [100], [011], [111] directions in germanium.

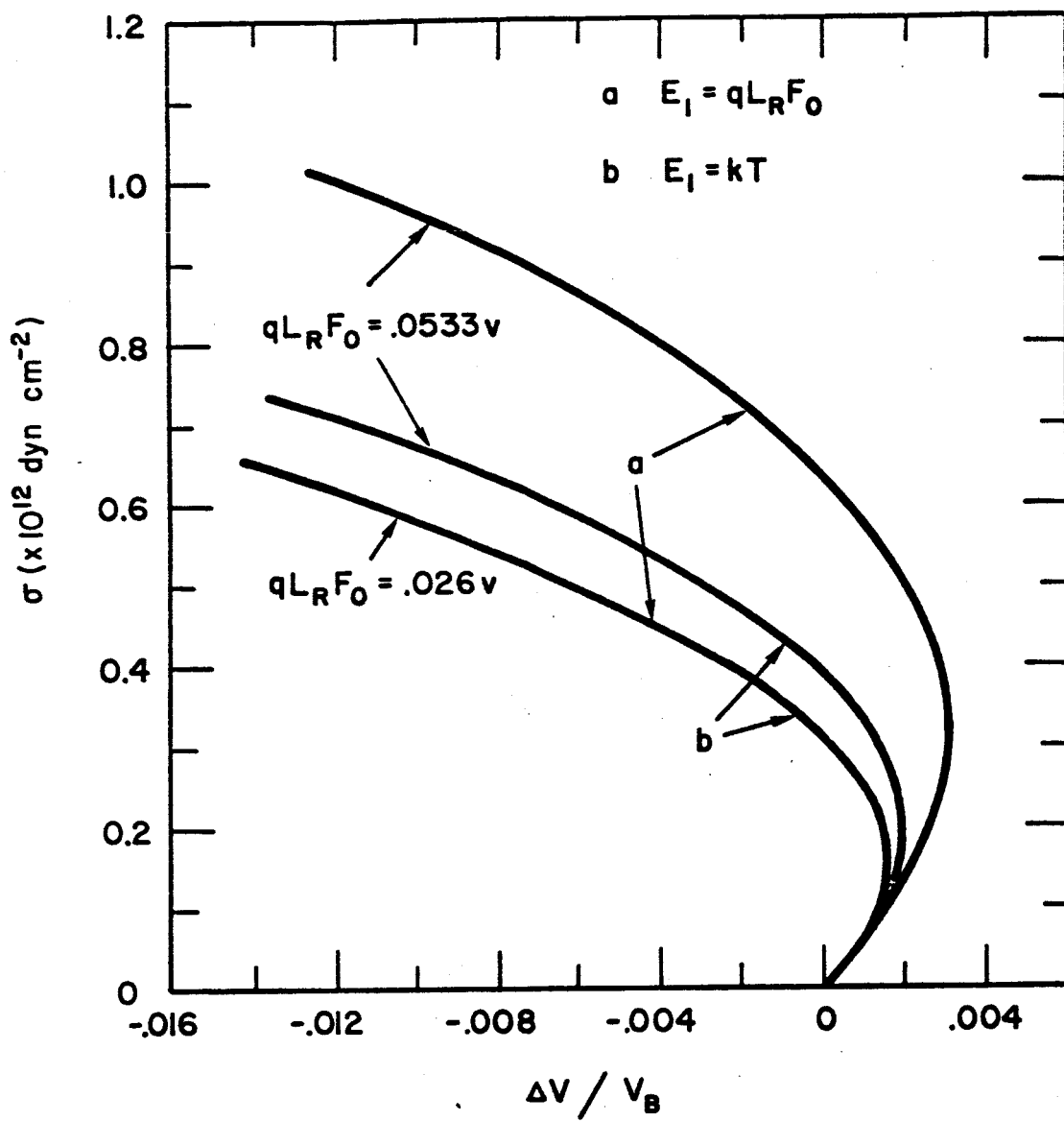


Figure 3. Comparison of electron breakdown in germanium for two values of E_1 (stress in [111] direction).

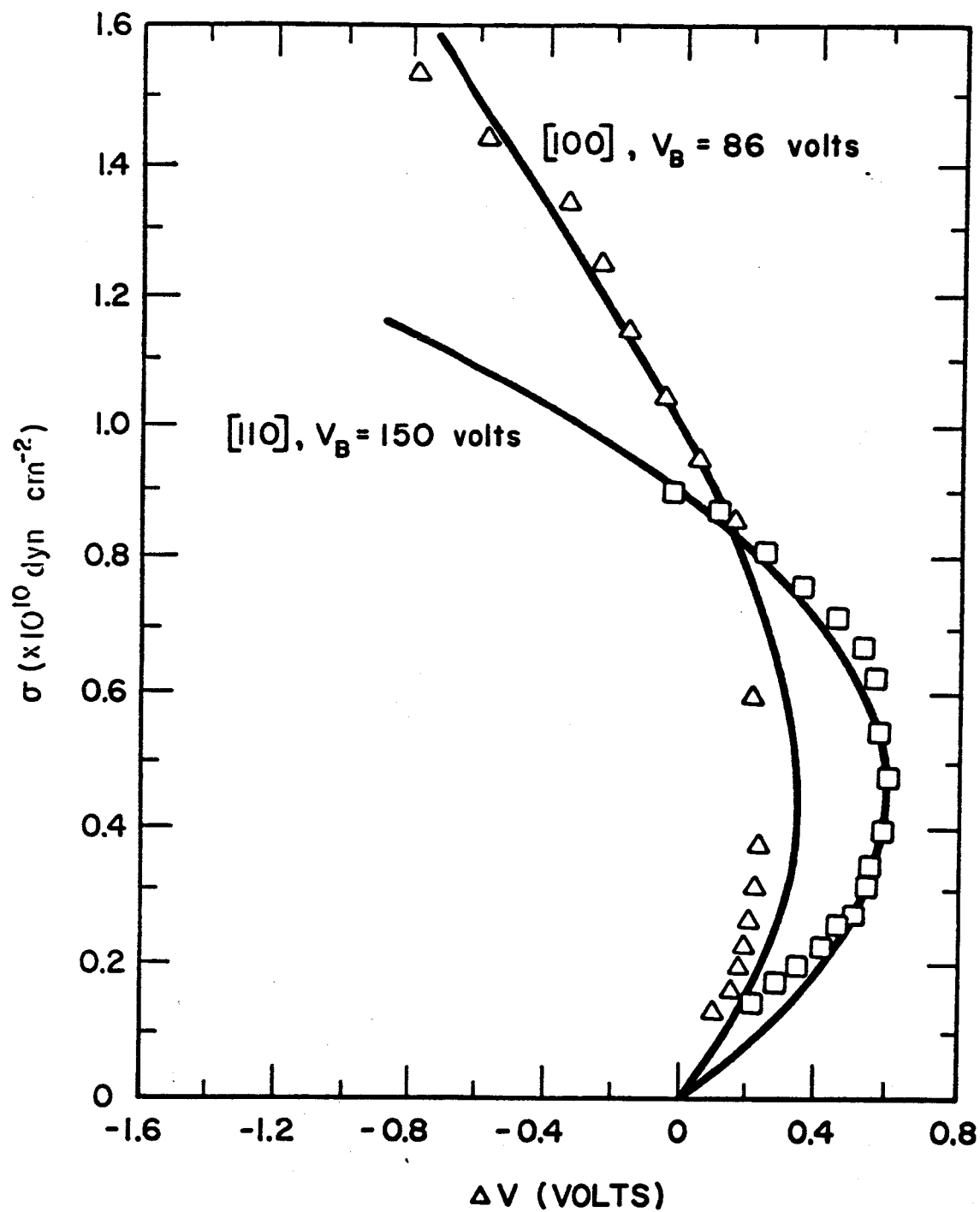


Figure 4. Change in breakdown voltage as a function of stress in germanium diodes. The solid lines are calculated values and the data points are experimental values reported by Rindner.⁷